Int. J. Heat Mass Transfer. Vol. 36, No. 2, pp. 315–323, 1993
Printed in Great Britain

0017-9310/93 $6.00+0.00

Analysis of Lewis number effects in flame spread

J. B. GREENBERG† and P. D. RONNEY
Department of Mechanical and Aerospace Engineering, Princeton University,
Princeton, NJ 08544, U.S.A.

(*Received* 30 *July* 1991 *and in final form* 17 *March* 1992)

Abstract—de Ris's formulation of the problem of flame spread over thin and thick solid fuel surfaces is extended to include the influence of nonunity Lewis numbers of oxidant and fuel vapors. A zeroth order formula for the flame spread velocity is deduced. To this order of approximation, the Lewis number of the oxidant affects the spread rate only through its effect on the flame temperature and the fuel Lewis number plays no role at all. Thin fuel predictions are compared with experiments on flame spread over thin paper samples in a variety of O_2-diluent atmospheres. Good agreement between theory and experiment supports the theory and indicates that unity Lewis number assumptions can lead to rather inaccurate spread rate and flame temperature predictions.

1. INTRODUCTION

THE STUDY of flame spreading over thin solid fuels provides a useful means for developing an understanding of the behavior of many nonpremixed flames, such as those to be found in building fires. The use of thin solid fuels simplifies the analysis of the spreading process as its enables heat conduction in the solid phase to be neglected; attention need only be focused on gas-phase conduction. de Ris [1] presented an analysis of a laminar diffusion flame spreading over a solid (or liquid) fuel bed against an airstream. The flame was assumed to heat the unburnt fuel bed which subsequently vaporized. The fuel vapors then reacted with the oxidant from the airstream, producing heat to maintain the process of flame spread.

A fundamental assumption of de Ris's analysis is that the Lewis numbers (*Le*) of the oxidizer and fuel are unity. This permitted the removal of nonlinear chemical source terms through employment of the usual Schwab–Zeldovich variables. de Ris obtained Weiner–Hopf integral equations that he solved approximately using a substitute kernel. He was then able to arrive at the following expression for the spread rate (v_f)

$$v_f = \frac{\sqrt{2}\lambda_g}{\rho_s C_{p,s} \tau_s} \frac{T_f - T_v}{T_v - T_\infty} \tag{1}$$

where λ_g, ρ, C_p, T and τ are the thermal conductivity, density, constant pressure specific heat, temperature and fuel bed thickness, respectively, and the subscripts g, s, f, v, ∞ refer to gas, solid fuel, flame, vaporization and ambient conditions, respectively. To interpret this result, one can note that (apart from the numerical factor $\sqrt{2}$) the numerator represents the rate of heat transfer in the gas phase to the fuel surface, whereas the denominator multiplied by the flame spread rate is the energy flux needed to vaporize the fuel surface. In de Ris's model the gas-phase chemistry was assumed to proceed at an infinite rate. No analytical solutions for the flame spread rate have been obtained for the case of finite rate gas-phase chemistry, although some numerical solutions have been reported [2, 3]. The absence of analytical solutions for flame spread with finite rate kinetics may not be a significant limitation, as the analysis of Linan [4] shows that for gaseous nonpremixed flames far from extinction, the burning process is mixing limited with finite rate chemistry, i.e. Damkohler number effects, playing an insignificant role.

Subsequent to de Ris's work, Thomas [5] examined the flame spread problem using heat transfer arguments only (mass transfer ignored). His simplified model led to the conclusion that the factor $\sqrt{2}$ in equation (1) should be replaced by $2/\pi$. The discrepancy between the two models seems to stem from the description of the upstream heat flux from the flame at the point where the flame meets the fuel surface. In de Ris's model its value is infinitely large whereas it has a finite value in Thomas's model. Subsequently, Wichman and Williams [6] reexamined this question, in view of spread rate–Damkohler number correlations of Fernandez-Pello *et al.* [7]. Using an analysis of the heat flux at the inception point they deduced that de Ris's model was a closer reproduction of the real situation than that of Thomas. Nevertheless, they suggested replacing the $\sqrt{2}$ factor in equation (1) by 0.8 so as to produce the correct spread rate in the finite Damkohler limit for the data of ref. [7]. Recently, an exact solution for the spread rate was given by

---

† On sabbatical leave from: Faculty of Aerospace Engineering, Technion, Israel Institute of Technology, Haifa 32000, Israel.

NOMENCLATURE

b	defined in equation (9d)
B	defined in equation (8a)
C_p	constant pressure specific heat
D	diffusion coefficient
h	heat of formation
K	defined in equation (8c)
l	length between surfaces in one-dimensional problem
L	heat of vaporization
Le	Lewis number
m	mass fraction
M	mass flux
Q	heat of reaction
S	chemical source term
T	temperature
v_a	opposed flow velocity
v_f	spread rate
x, y	Cartesian coordinates
X	mole fraction
y_f	flame location.

Greek symbols

ε	defined in equation (10)
η	defined in equation (8d)
θ_1, θ_2	coupling variables defined in equation (5)
λ	thermal conductivity
ν	stoichiometric coefficient
ρ	density
τ	thickness.

Subscripts

f	flame conditions
F	relating to fuel
g	gas phase conditions
i	relating to species i
o	relating to oxidant
s	solid fuel conditions
v	vaporization conditions
y	relating to the y-direction
∞	ambient conditions.

Delichatsios [8], based on de Ris's formulation of the problem. He found that for large values of $(T_f - T_v)/(T_v - T_\infty)$ the $\sqrt{2}$ factor in equation (1) should be replaced by $\pi/4$—a result close to the suggestion of Wichman and Williams [6].

For all the aforementioned analyses, unit Lewis numbers were assumed. However, Zhang *et al.* [9] recently reported an experimental investigation of flame spread over thin fuel samples in a variety of O_2–diluent atmospheres for which the Lewis numbers differ appreciably from unity. A comparison of their measured data with the predictions of equation (1) revealed a systematic error that correlated fairly well with the oxidant Lewis number. They therefore suggested incorporating into equation (1) the effect of Le_o on T_f, which they borrowed from an analysis of Law and Chung [10] for one-dimensional gaseous nonpremixed flames. The limit of small normal convective flux (relative to the diffusive flux) in the Law and Chung analysis was taken because the de Ris model is essentially a creeping flow problem [6]. This *ad hoc* modification to equation (1) provided a much better agreement between theory and experiment.

The experimental data and observations of Zhang *et al.* [9] provide the motivation for the current work. Here we shall present a more rigorous analysis of the flame spread problem for nonunity Lewis numbers of fuel and oxidant. A spread velocity formula (akin to equation (1)) will be given for this more general case. Interestingly, to the order of approximation employed, the effect on Le_o on flame spread rate is found to occur only through its effect on T_f, in concurrence with the heuristic approach of Zhang *et al.* Finally, comparisons with experimental data are made. These comparisons indicate a more subtle dependence of the flame spread velocity on Le_o through T_f than that considered by Zhang *et al.*

2. FORMULATION OF THE PROBLEM

2.1. *Governing equations*

The formulation of the flame spread problem that we present here essentially mimics that of de Ris [1], with the exception that we relax the unit Lewis number restriction. The flow configuration is shown in Fig. 1. As in de Ris [1], only heat and mass conservation equations are considered and all gas-phase properties are taken to be constant including the density. The species mass conservation equation reads (see Nomenclature)

$$\rho v_a \frac{\partial m_i}{\partial x} = \rho D_i \left[\frac{\partial^2 m_i}{\partial x^2} + \frac{\partial^2 m_i}{\partial y^2} \right] + S_i \qquad i = \text{fuel, oxidant} \quad (2a)$$

and energy conservation reads

$$\rho C_p v_a \frac{\partial T}{\partial x} = \lambda_g \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] + \sum_i h_i S_i \quad (2b)$$

(radiation effects have been omitted).

Now define the following nondimensional quantities (bearing asterisks)

$$\left. \begin{aligned} (x^*, y^*) &= (\rho v_a C_p/\lambda_g)(x, y) \\ m_o^* &= m_o(\nu_F M_F/\nu_o M_o) \quad T^* = (C_p/Q)T \\ S_F^* &= (\lambda_g/C_p)/(\rho v_a)^2 S_F \quad Le_i = \lambda_g/\rho C_p D_i \end{aligned} \right\} \quad (3)$$

in which the heat of reaction is given by

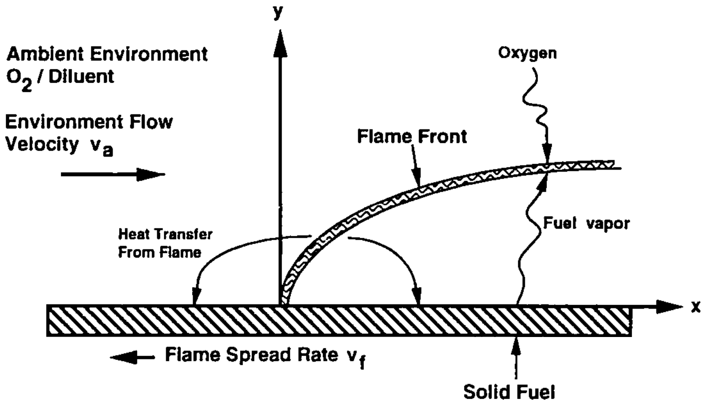

FIG. 1. Flow configuration for flame spread problem.

$$Q = \sum_i h_i S_i / S_F.$$

Rewriting equations (2) leads to their nondimensional form (omitting asterisks for the sake of convenience)

$$\frac{\partial m_F}{\partial x} = \frac{1}{Le_F}\left[\frac{\partial^2 m_F}{\partial x^2} + \frac{\partial^2 m_F}{\partial y^2}\right] - S_F \tag{4a}$$

$$\frac{\partial m_o}{\partial x} = \frac{1}{Le_o}\left[\frac{\partial^2 m_o}{\partial x^2} + \frac{\partial^2 m_o}{\partial y^2}\right] - S_F \tag{4b}$$

$$\frac{\partial T}{\partial x} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + S_F \quad (S_F \geqslant 0). \tag{4c}$$

Now we form the following Schwab–Zeldovich type of variables

$$\theta_1 = \hat{T} + \hat{m}_o/Le_o \quad \theta_2 = m_F/Le_F - \hat{m}_o/Le_o \tag{5}$$

in which

$$(\hat{T}, \hat{m}_o) = (T - T_\infty, m_o - m_{o\infty}). \tag{6}$$

Hence, combining equations (4) with the aid of equations (5) and (6) produces

$$\frac{\partial \theta_1}{\partial x} = \frac{\partial^2 \theta_1}{\partial x^2} + \frac{\partial^2 \theta_1}{\partial y^2} - \left(1 - \frac{1}{Le_o}\right)\frac{\partial \hat{m}_o}{\partial x} \tag{7a}$$

$$\frac{\partial \theta_2}{\partial x} = \frac{\partial^2 \theta_2}{\partial x^2} + \frac{\partial^2 \theta_2}{\partial y^2} - \left(1 - \frac{1}{Le_F}\right)\frac{\partial m_F}{\partial x} + \left(1 - \frac{1}{Le_o}\right)\frac{\partial \hat{m}_o}{\partial x}. \tag{7b}$$

The use of the functions θ_1, θ_2 ensures that one correctly describes the conditions at the infinite Damkohler number flame sheet that we assume here and the jump conditions across this sheet [11].

2.2. *Boundary conditions*

Referring to Fig. 1 the boundary conditions for $x \geqslant 0$, $y = 0$ are

$$\theta_1 = (T_v T_\infty) - m_{o\infty}/Le_o = -B \tag{8a}$$

$$\frac{\partial \theta_2}{\partial y} = -K\frac{\partial \theta_1}{\partial y} \tag{8b}$$

in which

$$K = \ln(1 + B/\eta)/B \tag{8c}$$

with

$$\eta = L/Q. \tag{8d}$$

We note that equation (8b) is a linearized boundary condition derived in a manner analogous to de Ris's original work [12] but, of course, including nonunity Lewis numbers through the definitions of θ_1 and θ_2. It predicts the correct mass transfer for the one-dimensional diffusion flame parallel to the vaporizing surface and provides good approximations to the flame temperature and position in the two-dimensional problem [1].

For the preheat zone ($x < 0$, $y = 0$), the boundary conditions are

$$b\frac{\partial}{\partial x}(\theta_1 + \theta_2) = \frac{\partial \theta_1}{\partial y} \tag{9a}$$

$$\frac{\partial \theta_2}{\partial y} = 0. \tag{9b}$$

Finally

$$\lim(\theta_1 + \theta_2) = T_v - T_\infty \tag{9c}$$

as $x \to 0$, $y \to 0^+$.

In the above

$$b \equiv \rho_s C_{p,s} \tau_s v_f / \lambda_g. \tag{9d}$$

Note also that:

$$\theta_1, \theta_2 \to 0 \quad \text{as} \quad y \to +\infty \quad \text{or} \quad x \to -\infty. \tag{9e}$$

3. SOLUTION OF THE PROBLEM

In order to extract an analytical solution from this formulation, we suppose that

$$\left|1-\frac{1}{Le_i}\right| = \varepsilon_i < 1. \tag{10}$$

Taking $Le_i = Le_o$ and examining typical values of the parameter ε_i for the oxidizing atmospheres used in the experiments of Zhang *et al.* [9], the validity of equation (10) is readily verified for most (but not all) cases. We shall exploit this small parameter to expand the dependent variables in series

$$(\theta_1, \theta_2, m_{\rm F}, m_{\rm o}) = \sum_{j=0}^{\infty} (\theta_1^{(j)}, \theta_2^{(j)}, m_{\rm F}^{(j)}, m_{\rm o}^{(j)})\varepsilon^j \tag{11a}$$

as well as the eigenvalue

$$v_{\rm f} = \sum_{j=0}^{\infty} v_{\rm f}^{(j)}\varepsilon^j \tag{11b}$$

where, for the sake of convenience we write $\varepsilon = \varepsilon_i = \varepsilon_{\rm o}$ for all *i*.

Substituting these expansions into equations (7a) and (7b) and the boundary conditions the following O(1) problem emerges:

$$\frac{\partial\theta_1^{(o)}}{\partial x} = \frac{\partial^2\theta_1^{(o)}}{\partial x^2} + \frac{\partial^2\theta_1^{(o)}}{\partial y^2} \tag{12a}$$

$$\frac{\partial\theta_2^{(o)}}{\partial x} = \frac{\partial^2\theta_2^{(o)}}{\partial x^2} + \frac{\partial^2\theta_2^{(o)}}{\partial y^2} \tag{12b}$$

with boundary conditions

$$\theta_1^{(o)} = -B \qquad x \leqslant 0,\ y = 0 \tag{13a}$$

$$\frac{\partial\theta_2^{(o)}}{\partial y} = -K\frac{\partial\theta_1^{(o)}}{\partial y} \qquad x \geqslant 0,\ y = 0 \tag{13b}$$

$$\frac{\partial\theta_2^{(o)}}{\partial y} = 0 \qquad x < 0,\ y = 0 \tag{13c}$$

$$b^{(o)}\frac{\partial}{\partial x}(\theta_1^{(o)}+\theta_2^{(o)}) = \frac{\partial\theta_1^{(o)}}{\partial y} \qquad x < 0,\ y = 0. \tag{13d}$$

It is evident that, *at least to this order of approximation*, the formulation of the nonunity Lewis number flame spread problem is *identical* to that of de Ris [1], with the exception of the parameter B in equations (13a) and (13b) that now contain a Lewis number, as do the 'coupling' functions θ_1, θ_2. We thus need only apply the ready-made results from the unit Lewis number problem and adjust them appropriately so as to obtain the O(1) value of $v_{\rm f}$ (or b). Delichatsios's [8] exact analysis for the flame spread velocity can be adopted, namely

$$b^{(o)} = \frac{\pi}{4}\left(\frac{T_{\rm f}^{(o)}-T_{\rm v}}{T_{\rm v}-T_{\infty}}\right) \tag{14}$$

with $T_{\rm f}$ borrowed from de Ris's expression (as Delichatsios's analysis does not provide the temperature or species concentration profiles)

$$T_{\rm f}^{(o)} = T_{\infty} + (m_{{\rm o}\infty}/Le_{\rm o})[1-B/\ln(1+B/\eta)]. \tag{15}$$

Examination of equations (14) and (15) reveals that the dependence of the flame spread rate on $Le_{\rm o}$ is more sophisticated than the expression suggested in ref. [9], however, their heuristic assumption that $Le_{\rm o}$ affects $v_{\rm f}$ only through its effect on $T_{\rm f}$ is supported by the current analysis. Note also that the Lewis number for the fuel vapors does not appear in equation (14) due to its fortuitous cancellation in the analysis (see also Appendix) which, in effect, results from the similar contributions of $Le_{\rm F}$ to the boundary conditions at the fuel surface and the flame sheet.

4. COMPARISON WITH EXPERIMENT DATA

Our O(1) solution for the flame spread rate, equation (14), was assessed by comparing its predictions against the experimental data of Zhang *et al.* [9], which to our knowledge is the only systematic study of diluent type (and thus Lewis number effects) on $v_{\rm f}$. Evaluation of the transport properties was performed using the recommendation of Wichman and Williams [6], namely to compute $\lambda_{\rm g}$ at the temperature $T_{\rm m} = (T_{\rm f}+T_{\infty})/2$. Accordingly, this value was also utilized for calculating the other relevant properties, namely C_p and $Le_{\rm o}$, for the sake of consistency. As in ref. [7], the values of $T_{\rm v}$, L, Q, $C_{p,{\rm s}}$, and $\nu_{\rm F}M_{\rm F}/\nu_{\rm o}M_{\rm o}$ were taken to be 618 K, 753 kJ kg^{-1}, 16 740 kJ kg^{-1}, 1.26 kJ kg^{-1} K^{-1} and 0.844, respectively, which are representative of cellulose. Also, for the fuel used in ref. [9], the fuel bed density per unit area ($\rho_{\rm s}\tau_{\rm s}$) was 0.0035 g cm^{-2}.

The agreement of equations (14) and (15) with experiments (not shown) was found to be rather good except for two situations: (1) near the extinction limit, where finite Damkohler number effects, not considered in our analysis, are present; and (2) when helium is the diluent. The latter is somewhat surprising in view of (i) the range of $Le_{\rm o}$ for the helium data ($1.08 \leqslant Le_{\rm o} \leqslant 1.42$, with the lower bound corresponding to $X_{{\rm O}_2} = 0.3$ where, surprisingly, the discrepancy was found to be largest); and (ii) the good agreement for the other diluents (including SF_6 and CO_2 for which the inequality in equation (10) is, at best, borderline). Because of (i) it is clear that inclusion of an O(ε) correction to the burning velocity is not likely to significantly improve matters.

Because of the discrepancy between theory and experiment in the case of helium, the following reassessment of the solution was carried out. In de Ris's ($Le_i = 1$) solution [1], the asymptotic flame temperature turns out to be the same as that of a one-dimensional model problem in which a diffusion flame is formed between two surfaces at $y = 0$ and 1. The lower surface is maintained at the vaporization temperature ($T_{\rm v}$) and fuel vapors are produced there. The upper surface is maintained at the oxidant–diluent composition and the ambient temperature. The solution of this model problem for a *general* Lewis number (see Appendix) yields a flame temperature

$$T_{\rm f,exact} = T_{\infty} + \{1-(1+m_{{\rm o}\infty})^{-1/Le_{\rm o}}\}\{T_{\rm v}-T_{\infty}+(1-\eta)\}. \tag{16}$$

It is significant that the ratio $\alpha = (T_{\mathrm{f,exact}}^{(o)} - T_{\infty})/(T_{\mathrm{f}} - T_{\infty})$ is in the range $0.9 \leqslant \alpha < 0.97$ for all the flames considered here, *with the exception of those having helium as a diluent* for which $0.8 \leqslant \alpha \leqslant 0.83$ because of the high $m_{\mathrm{o}\infty}$ of He-diluted atmospheres (due to the low molecular weight of He).

As a consequence of this significant overestimate of T_{f}, the spread rates were recomputed using equation (14) but substituting $T_{\mathrm{f,exact}}$ for $T_{\mathrm{f}}^{(o)}$. In dimensional variables this can be written as

$$v_{\mathrm{f}} = \frac{\pi}{4} \frac{\lambda_{\mathrm{g}}}{\rho_{\mathrm{s}} C_{p,\mathrm{s}} \tau_{\mathrm{s}}} \frac{T_{\mathrm{f,exact}} - T_{\mathrm{v}}}{T_{\mathrm{v}} - T_{\infty}} \tag{17a}$$

$$T_{\mathrm{f,exact}} = T_{\infty} + \left\{1 - \left(1 + \frac{m_{\mathrm{o}\infty} \nu_{\mathrm{F}} M_{\mathrm{F}}}{\nu_{\mathrm{o}} M_{\mathrm{o}}}\right)^{-1/Le_{\mathrm{o}}}\right\}\left\{T_{\mathrm{v}} - T_{\infty} + \frac{Q-L}{C_p}\right\}. \tag{17b}$$

In Figs. 2(a)–(f) we show a comparison between this 'corrected' spread rate expression with experimental data and computed values for which Le_{o} was forcibly set equal to unity. Again, only results for values of X_{O_2} away from the extinction limits are shown, since finite-rate chemistry effects will be present near extinction limits and these effects are not accounted for in our analysis. Furthermore, only relatively low values of X_{O_2} are shown because as $X_{\mathrm{O}_2} \to 1$, the effect of the different diluents and their respective Lewis numbers disappears. The corrected values shown in Fig. 2 are quite close to the experimental data and the Lewis number effect is very noticeable. The theoretical spread rate with $Le_{\mathrm{o}} = 1$ is greater/less than the theoretical spread rate with $Le_{\mathrm{o}} \neq 1$ depending on whether the oxidant Lewis number is greater/less than unity. This behavior is, of course, intimately tied in with the computed flame temperature (cf. equation (17)) which is greater/less than the $Le_{\mathrm{o}} = 1$ flame temperature when Le_{o} is less/greater than unity. This is to be expected considering the heat-to-mass diffusion ratio connotation of the Lewis number.

In order to provide a tangible indication of the range of flame temperatures and Lewis number of interest we list in Table 1, for representative flame spread conditions, computed values of these quantities as well as the falsified T_{f} obtained by setting $Le_{\mathrm{o}} = 1$. Particularly striking is the difference between $T_{\mathrm{f,exact}}$ and $T_{\mathrm{f}(Le=1)}$ for the case of SF_6 as the diluent. A factor of more than two provides a clear rationale for the discrepancy between the corresponding spread rate curves illustrated in Fig. 2(d). (While the values of $T_{\mathrm{f,exact}}$ are rather high in this case, it should be noted that dissociation at high temperatures, which would lower T_{f}, is not considered within this model. Zhang *et al.* [9] suggest a means to incorporate the effect of dissociation into the analytical estimate of T_{f} when $Le \neq 1$.)

In Fig. 3 we combine all the computed (using equation (17)) and experimental spread rate data to illustrate the Lewis number effect on spread rate. The inclined straight line is the best fit of the theoretical to experimental spread rate ratio *if one takes* $\mathrm{Le}_{\mathrm{o}} = 1$ *artificially* in equation (17). Our O(1) analysis of Lewis number effects is supported by the horizontal straight line fit through the $Le_{\mathrm{o}} \neq 1$ data at a value of almost one. These data clearly demonstrate that a unity Lewis assumption may lead to more than ±50% inaccuracy in predicting the spread rate.

In Table 2 we compare some measured flame temperatures with our theoretical predictions. The data were collected using thermocouple measurements cor-

Table 1. Representative calculated flame temperatures and Lewis numbers

Diluent	X_{O_2}	Le_{O_2}	$T_{\mathrm{f}}^{(o)}$ (K) (equation (15))	$T_{\mathrm{f,exact}}$ (K) (equation (17))	$T_{\mathrm{f,exact}}$ ($Le = 1$) (K)
He	0.30	1.09	3113	2550	3202
N_2	0.30	0.82	3198	2879	2504
CO_2	0.40	0.61	3752	3372	2413
SF_6	0.50	0.33	4267	4029	1822
Ne	0.30	1.12	3306	2915	3202
Ar	0.25	0.98	3351	3137	2804

Table 2. Comparison of measured and theoretical flame temperatures with ambient O_2 mole fraction = 0.2

Diluent	Le	Measured flame temperature (K)	Theoretical flame temperature (equation (17)) (K)	Theoretical flame temperature (equation (17), $Le = 1$) (K)
Ar	0.98	2000	2681	2601
He	1.37	1620	1684	2601
N_2	0.81	1720	2068	1948
Ne	1:25	1730	2170	2601

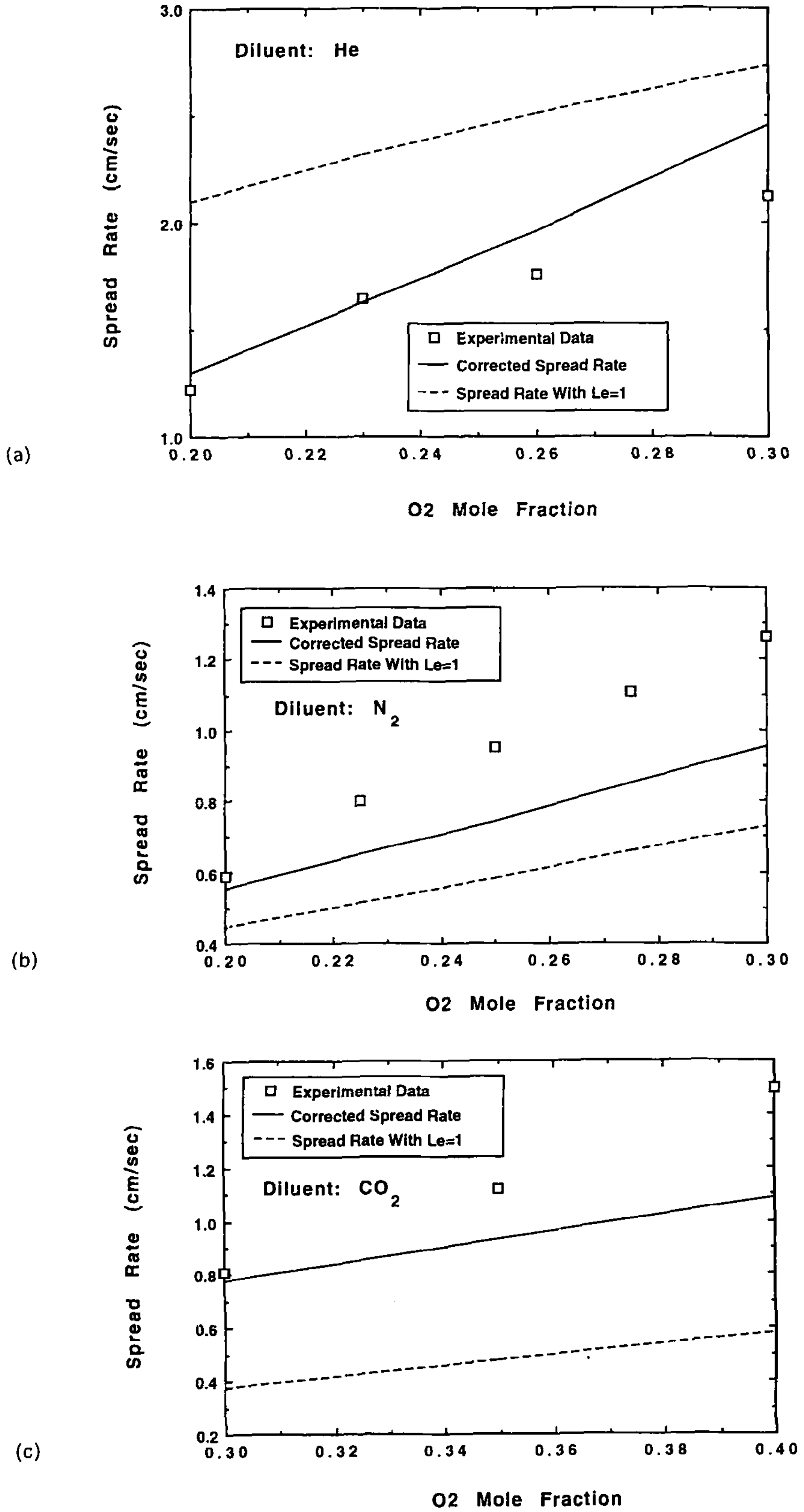

FIG. 2. Flame spread rate vs O_2 mole fraction for various diluents—comparison of theoretical predictions using equation (17), theoretical predictions using $Le_0 = 1$, and experimental data : (a) helium ; (b) nitrogen ; (c) carbon dioxide ; (d) sulphur hexafluoride ; (e) neon ; and (f) argon.

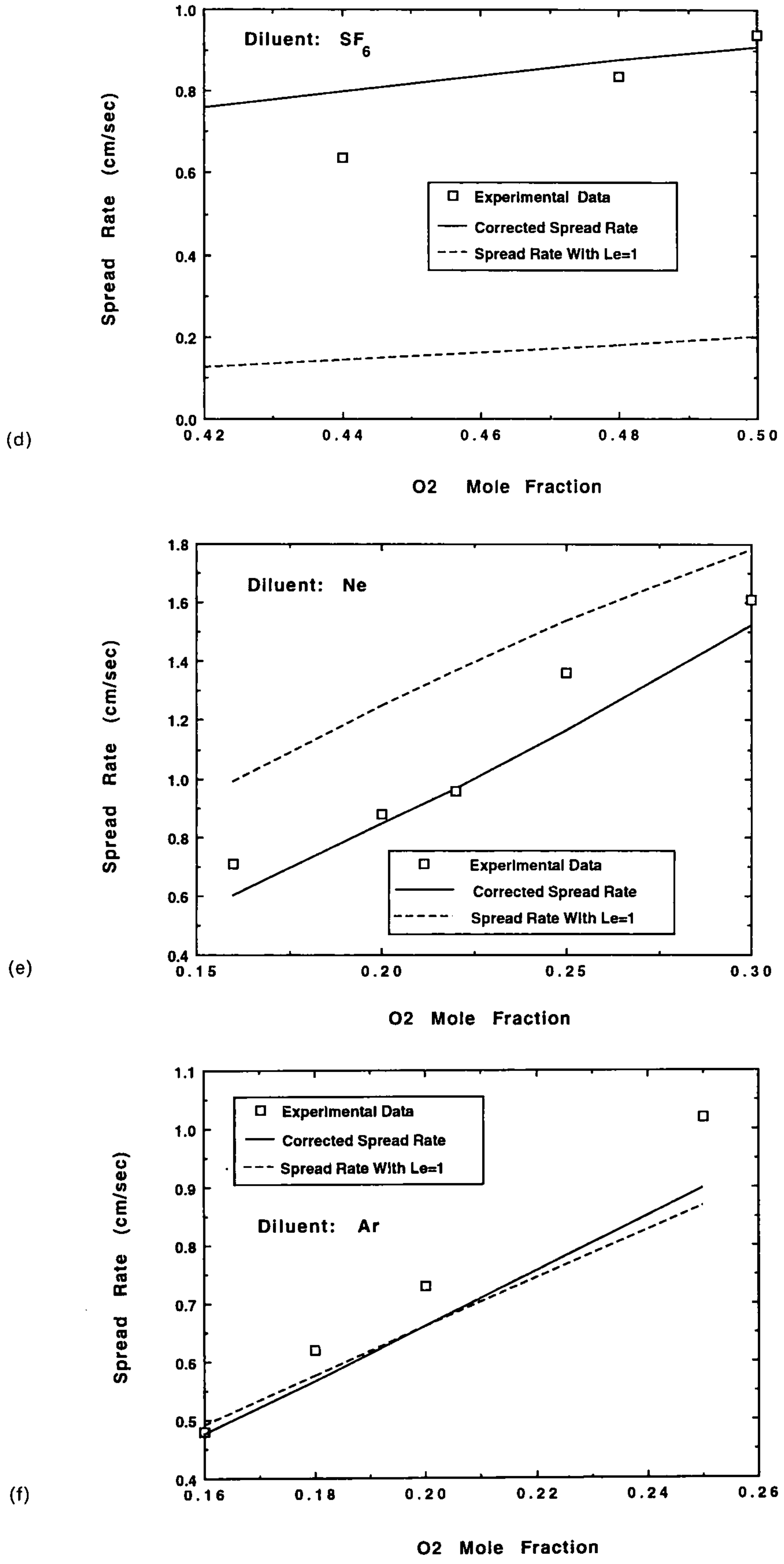

FIG. 2—*Continued.*

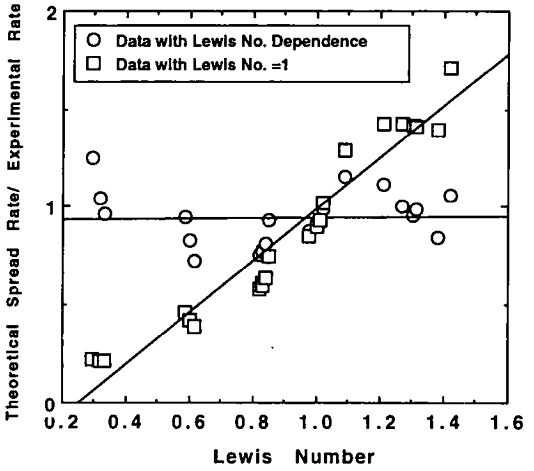

Fig. 3. Ratio of experimental and theoretical (using equation (17)) flame spread rates as a function of Lewis number with and without incorporation of Lewis number effects.

rected to account for radiative losses from the thermocouple bead. An ambient oxygen mole fraction of 0.2 was taken in each case. The overall *trend* is correct, although the apparent deviation of the theoretical results is a little surprising in view of the rather good agreement of the flame spread data. As discussed above, the discrepancy is probably due mainly to dissociation effects, these becoming more significant at higher flame temperatures.

5. THICK FUEL ANALYSIS

The analysis and comparison with experimental data presented here has focused on flame spread over *thin fuels*, thus enabling heat conduction in the solid phase to be neglected. It is interesting to note that de Ris's *thick fuel* analysis [1], in which heat conduction in the solid fuel *is* accounted for, can be extended to include nonunity Lewis number effects using exactly the same approach we have given here. In this case the zeroth order flame spread velocity is

$$v_{\mathrm{f}}^{(\mathrm{o})} = \left(\frac{\rho C_p \lambda_{\mathrm{g}} v_{\mathrm{a}}}{\rho C_{p,s} \lambda_{\mathrm{s},y}}\right)\left(\frac{T_{\mathrm{f}}^{(\mathrm{o})} - T_{\mathrm{v}}}{T_{\mathrm{v}} - T_{\infty}}\right)^2 \tag{18}$$

in which $\lambda_{s,y}$ is the thermal conductivity of the solid fuel in the y-direction (i.e. normal to its surface). The Le_{o} dependence of the flame spread velocity enters via $T_{\mathrm{f}}^{(\mathrm{o})}$ (and hence also indirectly through ρ, C_p, λ_{g}) and is stronger than in the thin fuel analysis because of the appearance of the square of the enthalpy ratio term. We note also that a similar leading order enthalpy squared relationship was given by Wichman *et al.* [13], based on overall energy balance considerations. We are not aware of any experimental studies of flame spreading over thick solid fuels in nonstandard atmospheres which might be used to test equation (18).

6. CONCLUSIONS

We have extended de Ris's analyses for the rate of flame spread over thin and thick solid fuels to include effects of the nonunity Lewis number of the oxidizing environment. A formula for flame spread is deduced and is enhanced using a heuristically deduced correction to the asymptotic flame temperature. The theory compares very well with available experimental data for a variety of oxidizing environments having Lewis numbers in the range $0.3 \leqslant Le_{\mathrm{o}} \leqslant 1.55$. The Le_{o} effect is rather marked and provides a clear indication that the often-used unit Lewis number assumption may lead to rather erroneous predictions.

Acknowledgement—This work was supported by the National Science Foundation Presidential Young Investigator Program, Grant No. CBT-8657228.

REFERENCES

1. J. N. de Ris, Spread of a laminar diffusion flame, *Twelfth Symp. (Int.) Combust.*, pp. 241–252. Combustion Institute, Pittsburgh (1969).
2. M. Sibulkin, Free convection diffusion flames from burning solid fuels, *Prog. Energy Combust. Sci.* **14**, 195–212 (1988).
3. S. Bhattarcharjee, R. A. Altenkirch, N. Srikantaiah and M. Vedha-Nayagan, A theoretical description of flame spreading over solid combustibles in a quiescent environment at zero gravity, *Combust. Sci. Technol.* **69**, 1–15 (1990).
4. A. Linan, The asymptotic structure of counterflow diffusion flames for large activation energies, *Acta Astronautica* **4**, 1007–1039 (1974).
5. P. H. Thomas, Thermal conduction from a moving half plane heat source to a nearby cold surface, *Combust. Sci. Technol.* **28**, 173–175 (1982).
6. I. S. Wichman and F. A. Williams, Comments on rates of creeping spread of flames over thermally thin fuels, *Combust. Sci. Technol.* **33**, 207–214 (1983).
7. A. C. Fernandez-Pello, S. R. Ray and I. Glassman, Flame spread in an opposed flow: the effect of ambient oxygen concentration, *Eighteenth Symp. (Int.) Combust.*, pp. 579–589. Combustion Institute, Pittsburgh (1981).
8. M. A. Delichatsios, Exact solution for the rate of creeping flame spread over thermally thin materials, *Combust. Sci. Technol.* **44**, 257–267 (1986).
9. Y. Zhang, P. D. Ronney, E. V. Roegner and J. B. Greenberg, Lewis number effects on flame spreading over thin solid fuels, *Combust. Flame* **90**, 71–83 (1992).
10. C. K. Law and S. H. Chung, Steady state diffusion flame structure with Lewis number variations, *Combust. Sci. Technol.* **29**, 129–145 (1982).
11. S. H. Chung and C. K. Law, On the flame-sheet assumption and flame temperature determination in combustion modelling, *Combust. Sci. Technol.* **35**, 297–310 (1984).
12. J. N. de Ris, The spread of a diffusion flame over a combustible surface, Ph.D. Thesis, Harvard University (1968).
13. I. S. Wichman, F. A. Williams and I. Glassman, Theoretical aspects of flame spread in an opposed flow over flat surfaces of solid fuels, *Nineteenth Symp. (Int.) on Combust.*, pp. 835–845. Combustion Institute, Pittsburgh (1982).

APPENDIX. DERIVATION OF FLAME TEMPERATURE, EQUATION (16)

We consider the following model one-dimensional diffusion flame problem. At $y = 0$ there is a vaporizing surface at temperature $T = T_v$. At $y = l$ ambient conditions are maintained for the oxidant–diluent composition and temperature. Using the same normalized quantities as in equation (3) together with $l = \lambda_g/\rho v_a C_p$ the governing non-dimensional equations for this problem are

$$M\frac{dm_F}{dy} - \frac{1}{Le_F}\frac{d^2m_F}{dy^2} = -S_F \tag{A1}$$

$$M\frac{dm_o}{dy} - \frac{1}{Le_o}\frac{d^2m_o}{dy^2} = -S_F \tag{A2}$$

$$M\frac{d\hat{T}}{dy} - \frac{d^2\hat{T}}{dy^2} = -S_F. \tag{A3}$$

In the above M is the unknown normalized constant mass flux. The boundary conditions are:

(i) at $y = 0$:

$$M = Mm_F - \frac{1}{Le_F}\frac{dm_F}{dy} = \frac{1}{\eta}\frac{dT}{dy} \tag{A4a}$$

$$m_o = 0 \tag{A4b}$$

$$\hat{T} = T_v - T_\infty \tag{A4c}$$

(ii) at $y = 1$:

$$m_F = 0 \tag{A5a}$$

$$m_o = m_{o\infty} \tag{A5b}$$

$$\hat{T} = 0. \tag{A5c}$$

Under the assumption of a flame sheet model the conditions at the flame front are:

(iii) at $y = y_f$:

$$\left[\frac{1}{Le_F}\frac{dm_F}{dy}\right]_{y_f^-}^{y_f^+} = \left[\frac{1}{Le_o}\frac{dm_o}{dy}\right]_{y_f^-}^{y_f^+} \tag{A6a}$$

$$\left[\frac{d\hat{T}}{dy}\right]_{y_f^-}^{y_f^+} = \left[-\frac{1}{Le_F}\frac{dm_F}{dy}\right]_{y_f^-}^{y_f^+} \tag{A6b}$$

$$m_F = m_O = 0. \tag{A6c}$$

These conditions are sufficient to supply the profiles of m_F, m_o, $\hat{T}$ between $y = 0, 1$ as well as the flame location, y_f, the flame temperature, T_f, and the mass flux, M. Solution of this problem is straightforward and yields

$$\left.\begin{aligned} m_F(y) &= \left\{\frac{T_v - T_f}{\eta(1 - e^{My_f})}\right\}\{1 - e^{Le_F M(y - y_f)}\} & 0 \leqslant y \leqslant y_f \\ &= 0 & y_f \leqslant y \leqslant 1 \end{aligned}\right\} \tag{A7}$$

$$\left.\begin{aligned} m_o(y) &= \frac{m_{o\infty}}{(e^{Le_o My_f} - e^{Le_o M})}\{e^{Le_o My_f} - e^{Le_o My}\} & y_f \leqslant y \leqslant 1 \\ &= 0 & 0 \leqslant y \leqslant y_f \end{aligned}\right\} \tag{A8}$$

$$\left.\begin{aligned} \hat{T}(y) &= (1 - e^{My_f})^{-1}\{(T_f - T_\infty) - e^{My_f}(T_v - T_\infty) \\ &\quad + (T_v - T_f)e^{My}\} \quad 0 \leqslant y \leqslant y_f \\ &= \frac{(T_f - T_\infty)}{(e^M - e^{My_f})}\{e^M - e^{My}\} \quad y_f \leqslant y \leqslant 1 \end{aligned}\right\} \tag{A9}$$

where

$$M = y_f^{-1}\ln\left[1 - \left(\frac{T_v - T_f}{\eta}\right)\right] \tag{A10}$$

$$y_f = \left[1 + Lc_o^{-1}\ln(1 + m_{o\infty})\Big/\ln\left[1 - \left(\frac{T_v - T_f}{\eta}\right)\right]\right]^{-1} \tag{A11}$$

$$T_f = T_\infty + \{1 - (1 + m_{o\infty})^{-1/Le_o}\}\{T_v - T_\infty + (1 - \eta)\}. \tag{A12}$$

T_f of equation (A12) is to be identified with $T_{f,\text{exact}}$ in equation (16). A remarkable feature of this expression for the flame temperature is that the fuel Lewis number, Le_F, does not appear in it due to cancellation upon applying boundary conditions (see also Chung and Law [11]). This may provide a rationale for the fact that experimental data on spread rates appear to match theoretical predictions, *irrespective of the type of thin fuel utilized.*

Finally we point out that if this model one-dimensional diffusion flame problem is solved for small mass flux, M, the equivalence of the linearized flame temperature, $T_f^{(0)}$, and the downstream asymptotic flame temperature, $T_{f,\text{exact}}$, can be established in a manner identical to de Ris's $Le_o = 1$ analysis [12]. This provides further support for replacing $T_f^{(0)}$ with $T_{f,\text{exact}}$ in the flame spread formula, equation (14).